Table 2 shows the results of the different possible methods of compression for the third-body series and for the J_2 series. Compression was not continued for the third body methods that generated 21,341 unique d's, as this automatically requires 21,341 multiplications, a far larger number than the other two methods. Note that for the J_2 series, a case occurs with a shorter final series length but a larger number of multiplications and additions. Note also that the smallest number of multiplications and additions requires the route DC:DF:FE for the third-body series, but requires the route DC:EC:EF for the J_2 series.

Conclusion

Which route of compression to take depends upon the particular series being compressed and upon the relative speeds of execution of multiplication and addition of the target computer. Also, there is normally no a priori method for knowing which of the four routes is optimal for a particular series, so all four should be tried. For these reasons, this technique is not intended to be used as normal output from an algebraic manipulator package, but rather as a step taken when implementing a theory in an application where run time is critical. Not every Poisson series will display the degree of compression found in this example. It is probable that every series will benefit to some extent. In particular, series generated using expansions are highly likely to compress by a significant amount. In addition, any series would benefit by the use of trigonometric identities, rather than system calls.

References

[1]Shelus, P.J. and Jefferys, W.H., "A Note on an Attempt at More Efficient Poisson Series Evaluation," *Celestial Mechanics,* Vol. 11, 1975, pp. 75-78.

[2]Coffey, S. and Deprit, A., "Fast Evaluation of Fourier Series," *Journal of Astronomics and Astrophysics,* Vol. 81, 1980, pp. 310-315.

[3]Kaufman, B. and Dasenbrock, R.R., "Semianalytic Satellite Theory for Long-Term Behavior of Earth and Lunar Orbiters," *Journal of Spacecraft and Rockets,* Vol. 10, June 1973, pp. 377-383.

[4]Cefola, P., Green, A., McCalin, W., Early, L., Proulx, R., and Taylor, S., "Semianalytic Satellite Theory – Application to Orbit Determination," AIAA Paper 80-1677, Aug. 1980.

[5]Gooding, R.H., "On the Generation of Satellite Position (and Velocity) by a Mixed Analytical-Numerical Procedure," *Advanced Space Research,* Vol. 1, 1981, pp. 83-93.

[6]Kaufman, B., "First Order Semianalytic Satellite Theory with Recovery of the Short Period Terms Due to Third Body and Zonal Perturbations," *Acta Astronautica,* Vol. 8, 1981, pp. 611-623.

[7]Dasenbrock, R.R., "A FORTRAN-Based Program for Computerized Algebraic Manipulation," Naval Research Laboratory, Washington, D.C., Rept. 8611, Sept. 1982.

Vertical Ascent to Geosynchronous Orbit

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Introduction

THE Space Shuttle was developed to provide transportation from Earth to low-Earth orbit (LEO), with transportation to geosynchronous Earth orbit (GEO) by orbit transfer vehicles. Since the final destination of many payloads is GEO, there is some incentive to consider vehicles that are not constrained to go to LEO on the way. Because a destination in GEO is always directly above a point on the equator, it is possible to ascend by going straight up. This approach is called "vertical ascent."

The purpose of this study was to examine vertical ascent to GEO and compare it to the more conventional approach. Trajectories were calculated using the Program to Optimize Simulated Trajectories (POST).[1] The chemical propulsion used represented a Space Shuttle Main Engine (SSME) with a two-position nozzle. At sea level, with an expansion ratio of 40, this engine provides 2.23 MN thrust with an exit area of 2.15 m^2 and a vacuum specific impulse of 446 s. Above 10 km, the nozzle was extended to an expansion ratio of 150, and the engine provided 2.32 MN thrust with an exit area of 8.06 m^2 and a vacuum specific impulse of 463 s.

Vertical Ascent Trajectory

A vertical ascent trajectory is illustrated in Fig. 1 for a case with all chemical rocket propulsion. The initial acceleration in multiples of Earth's gravity a_0 is 1.3, and the maximum a_m is 3.0. In the calculations the radial velocity was approximately 20 m/s at GEO.

The thrusting strategy is illustrated in Fig. 2 for a gross mass of 1.5 Gg. As the vehicle rises, the thrust increases as the atmospheric pressure falls. At an altitude of 10 km, the nozzle exension was deployed. At about 70 km, the thrust has reached the vacuum thrust level and is nearly constant. As the vehicle mass is reduced by propellant consumption, the acceleration reaches the selected maximum. After this point, the thrust is reduced continuously such that the acceleration remains at a_m.

The thrust angle θ is vertical initially and decreases slightly as the vehicle rises. The eastward thrust component accelerates the vehicle just enough to keep the vehicle above the launch site. At some point in the ascent, the vertical thrust component is no longer needed. For the case shown, this point is at an altitude of 1662 km. Above this altitude, θ is zero. Since only a small eastward thrust is required, the thrust level drops at this point.

For the vertical ascent with $a_0 = 1.3$ and $a_m = 3.0$, the total ideal velocity increment (ΔV) is 17.7 km/s. Of the total, only about 3.5 km/s are required in the dense atmosphere, which ends at about 100 km.

Comparison with Hohmann Ascent

To provide a basis for comparison for the vertical ascent, Hohmann ascent trajectories were calculated with the same propulsion characteristics. The insertion perigee was 92.6 km. Several values of a_0 and a_m were considered for both Hohmann and vertical ascent. The results (see Fig. 3) indicate that the Hohmann ascent requires a significantly lower ΔV, about 12.8 km/s rather than 17.7 km/s for vertical ascent.

Advanced Propulsion

If chemical rockets are used for the initial part of the vertical ascent and an advanced propulsion system, such as a laser system, is used for the remainder, the performance may be improved because of the higher specific impulse of the advanced system. Laser propulsion may be achieved more easily with a vertical ascent because the laser beam could always be pointed along the path from the launch site to the destination. In Fig. 4, some estimates for the ratio of the initial mass to the final mass R are shown to provide some insight into the potential of vertical ascent with advanced propulsion. The results are based on ideal rocket calculations.

The lowest curve in Fig. 4 shows how R increases with ΔV for a chemical rocket. A mass ratio of 17 corresponds to the ΔV for a Hohmann ascent of 12.8 km/s. The vertical ascent mass ratio is off the scale at 50 for a ΔV of 17.7 km/s with

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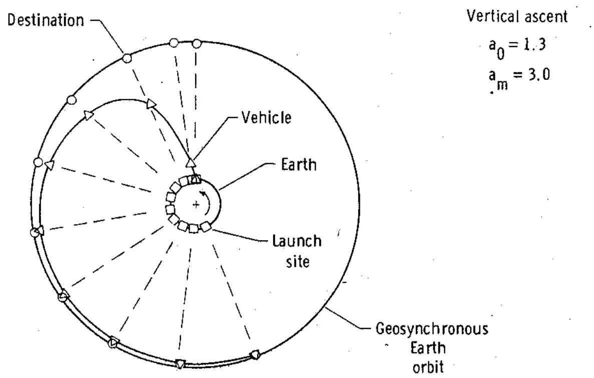

Fig. 1 Vertical ascent as seen from above the North Pole.

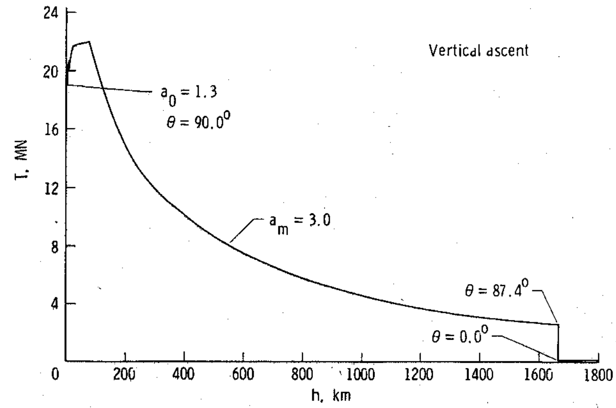

Fig. 2 Thrust strategy.

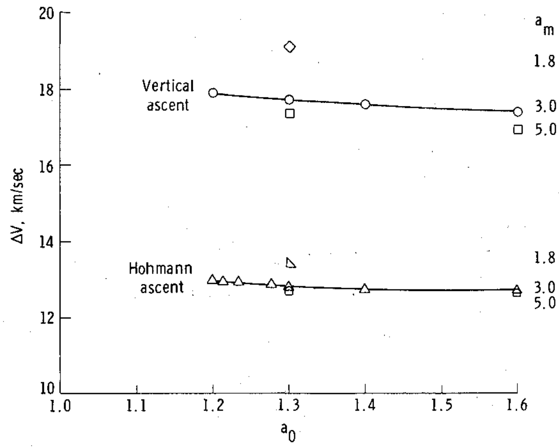

Fig. 3 Ideal velocity comparisons.

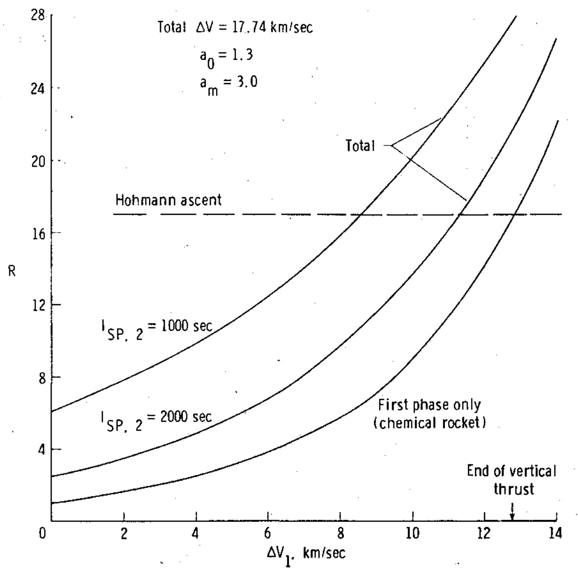

Fig. 4 Effect of laser propulsion on ideal velocity.

chemical-rocket propulsion. For values of R greater than about 10, staged vehicles would be needed.

If advanced propulsion with a specific impulse of 1000 s is used for the final portion of the vertical ascent, the total mass ratio is given by the upper curve. The total mass ratio is the product of the mass ratio for the first phase and the mass ratio for the second phase. The mass ratio for the second phase is 6.1 at a ΔV_l of zero (advanced propulsion only) and decreases to zero at a ΔV_l of 17.7 km/s (chemical-rocket propulsion only). At a ΔV_l of about 8 km/s, the total mass ratio for vertical ascent with some advanced propulsion is about the same as the mass ratio for a Hohmann ascent with only chemical-rocket propulsion. Therefore, there is no advantge from the vertical ascent with some advanced propulsion if ΔV_l is 8 km/s or more. If ΔV_l is less than 8 km/s, there is a possible advantage from the vertical ascent with some advanced propulsion. In fact, if the advanced propulsion is used for the entire exoatmospheric portion of the flight, the advantage may be significant. The ΔV required before the vehicle leaves the dense portion of the atmosphere is only about 3.5 km/s. At a ΔV_l of 3.5 km/s, the total mass ratio is only about 9, which is nearly low enough to consider using a single-stage vehicle for Earth-to-GEO transportation.

If the advanced propulsion system has a specific impulse of 2000 s, the total mass ratio is given by the middle curve of Fig. 4. At the lower values of ΔV_l, the increased specific impulse reduces R by over 50%. At a ΔV_l of 3.5, the total mass ratio is only 4.5, and single-stage vehicles definitely could be considered.

On Fig. 4, the point at which the vertical thrust ends is noted at a ΔV_l of 12.7 km/s. If chemical-rocket propulsion could be used for all of the vertical thrust, which requires a high thrust level, the advanced propulsion system could be quite small. Unfortunately, the ΔV_l of 12.7 km/s is itself almost the same as the ΔV for an entire Hohmann ascent, so the advanced propulsion system must provide some of the vertical thrust for the concept to be attractive.

Figure 4 was prepared based on a ΔV of 17.7 km/s, the correct value for the case with only chemical-rocket propulsion. Changing the thrust history would change the ΔV. Switching to the advanced propulsion system at a ΔV_l of 3.5 km/s at an altitude of 100 km would require a very high thrust from the advanced propulsion system for the thrust history shown in Fig. 2. A more optimal approach would be to use some lower thrust for the advanced propulsion system. A complete vehicle analysis would be required to find the optimum thrust.

Concluding Remarks

A preliminary investigation was conducted into vertical ascent from Earth to geosynchronous Earth orbit by ascending while always remaining above the launch site. Such a scheme might be useful if laser propulsion were used. The results indicate that with advanced propulsion vertical ascent could be of interest.

Reference

[1] Brauer, G.L., Cornick, D.E., and Stevenson, R., "Capabilities and Applications of the Program to Optimize Simulated Trajectories (POST)," NASA CR-2770, Feb. 1977.